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S-matrix elements and off-shell tachyon action with non-abelian gauge symmetry

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Abstract: We propose that there is a unique expansion for the string theory S-matrix elements of tachyons that corresponds to non-abelian tachyon action. For those S-matrix elements which, in their expansion, there are the Feynman amplitudes resulting from the non-abelian kinetic term, we give a prescription on how to find the expansion. The gauge invariant action is an α' expanded action, and the tachyon mass m which appears as coefficient of many different couplings, is arbitrary.
We then analyze in details the S-matrix element of four tachyons and the S-matrix element of two tachyons and two gauge fields, in both bosonic and superstring theories, in favor of this proposal. In the superstring theory, the leading terms of the non-abelian gauge invariant couplings are in agreement with the symmetrised trace of the direct non-abelian generalization of the tachyonic Born-Infeld action in which the tachyon potential is consistent with $V(T) = e^{\pi\alpha' m^2 T^2}$. In the bosonic theory, on the other hand, the leading terms are those appear in superstring case as well as some other gauge invariant couplings which spoils the symmetrised trace prescription. These latter terms are zero in the abelian case.

Keywords: Bosonic Strings, D-branes, Tachyon Condensation.

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Contents

1. The idea

Decay of unstable branes is an interesting process which might shed new light in understanding properties of string theory in time-dependent backgrounds [1]–[29]. In particular, by studying the unstable branes in the boundary conformal field theory (BCFT), Sen has shown that the end of tachyon rolling in this theory is a tachyon matter with zero pressure and non-zero energy density [3]. These results is then reproduced in field theory by the tachyonic DBI action [30]–[32]. The form of tachyon potential in this action at minimum of potential is then fixed by using the fact that the higher derivative corrections to the tachyonic DBI action are not important at this point, and the action should not have plane wave solution [4]. Possible application of the tachyonic DBI action to cosmology have been discussed in [33].

The tachyonic DBI action was originally proposed as an action that when transformed under the Seiberg-Witten map, i.e., a non-commutative gauge invariant action, is consistent with the non-commutative expansion of the S-matrix element of one graviton and two tachyons in the presence of background B-flux [30], i.e., an expansion around $s = 0$ which produces massless pole as first leading term and contact term with the $*'$-product as next leading term.[1] The massless pole is reproduced by non-commutative auge invariant kinetic term, and the contact term is reproduced by the non-commutative tachyon-tachyon-graviton coupling. Both exist in the above non-commutative action.[2] The Mandelstam

[1] The $*'$-product reflects the fact that the non-commutative gauge invariant coupling of graviton to D-brane has open Wilson line [34]–[43].

[2] Throughout the paper, we are appealing to the specific meaning of the tachyon action as a generating functional for producing the leading terms of the string theory S-matrix elements.

JHEP12(2003)036

variable s in the amplitude is arbitrary, however, expansion around any other point is not consistent with the non-commutative gauge symmetry, e.g., it does not produce the massless pole, nor the contact term with $*'$-product. We speculate that this is a general fact for all other S-matrix elements involving tachyons. That is, there is only one limit for the S-matrix elements that is consistent with non-commutative gauge symmetry. We call this limit the non-commutative limit. Expansion of S-matrix elements at this limit would produce the Feynman amplitudes that result from the non-commutative gauge invariant tachyon action.

When there are N coincident D-branes the U(1) gauge symmetry of an individual D-brane is enhanced to a non-abelian U(N) symmetry [49]. The above proposal in this case is that there is only one limit for the S-matrix elements involving tachyons that is consistent with the non-abelian gauge symmetry. We call this limit the non-abelian limit. The limit would be such that when the S-matrix elements expanded around it, they reproduce the Feynman amplitudes resulting from non-abelian gauge invariant tachyon action. In general, however, it is nontrivial to find such a limit for a S-matrix element.

For a class of S-matrix elements this limit can be found easily. This class includes those S-matrix elements that produce, at the non-abelian limit, massless and/or tachyonic poles which are related to the non-abelian kinetic term in field theory side, e.g., the above example [30] in the non-commutative case. Since both tachyon and massless transverse scalars transform in the adjoint representation of U(N) group, they have identical non-abelian kinetic terms. The Feynman amplitudes resulting from kinetic terms are, then, similar in both cases. Therefore, the expansion of S-matrix element of tachyons and the expansion of S-matrix element of scalars should be similar to produce these Feynman amplitudes. On the other hand, we know the non-abelian limit of the S-matrix element of the scalars, i.e., sending all Mandelstam variables to zero. Using similar steps for the S-matrix element of tachyons, one would find the non-abelian/non-commutative limit of the S-matrix element of tachyons. The non-commutative limit of the S-matrix element of four tachyon vertex operators has been found in [55] in this way.

An interesting observation, made in [56, 57], in the non-commutative case, is that the S-matrix element of tachyon vertex operators and the S-matrix element of the massless scalar vertex operators can be written in a universal/off-shell form. The non-commutative limit of the S-matrix elements in the universal form is the limit that all the Mandelstam variables in the amplitude approach zero. This observation can be made in the non-abelian case as well. We interpret this, here, as an indication that the on-shell mass of tachyon or the scalar does not appear in the S-matrix element in the universal from. Accordingly, the tachyon mass in the non-abelian action must be arbitrary/off-shell. The non-abelian tachyon action is an action expanded in terms of α' which includes covariant derivative of tachyon and the off-shell mass. The off-shell mass may appears as coefficient of many different terms in the action. If one restricts the arbitrary mass to the on-shell value of tachyon, i.e., $m^2 \sim -1/\alpha'$, then, the expansion is not an α' expansion any more. Clearly, for the on-shell massless scalar case, the action would be the non-abelian low-energy-effective action of the D-branes.

JHEP12(2003)036

In this paper, we would like to analyze in details the S-matrix element of four tachyons, and the S-matrix element of two tachyon and two gauge field vertex operators in favor of the above proposal. We perform this analyzes in both superstring and bosonic string theory. In the superstring case that we perform the calculations in the next section 2, we show that the S-matrix element of four tachyons in the non-abelian limit has massless poles, and infinite tower of contact terms. And the S-matrix elements of two tachyons and two gauge fields has both tachyonic and massless poles, and infinite tower of contact terms. The tachyonic and massless poles are reproduced in field theory side by the Feynman amplitudes resulting from non-abelian kinetic term. The leading contact terms, on the other hand, are reproduced by some other non-abelian gauge invariant couplings. We will show that these couplings are fully reproduced by expansion of the symmetrised trace of the non-abelian tachyonic BI action with tachyon potential $V(T) = e^{\pi\alpha' m^2 T^2}$. In section 3, we perform the calculations for the bosonic case. The amplitudes in this case has two kind of tachyonic poles. In the non-abelian limit, one of them must be expanded and the other one is reproduced by the standard non-abelian kinetic term. In this case, the leading contact terms are consistent with the above non-abelian tachyonic BI action, and some other gauge invariant couplings which are zero in the abelian case. Section 4 is devoted for some discussions on the results.

2. Amplitudes in superstring theory

Using the world-sheet conformal field theory technique [45], one can evaluate any 4-point functions by evaluating the correlation function of their corresponding vertex operators. Performing the correlators, one finds, in general, that the integrand has $\mathrm{SL}(2,\mathbb{R})$ symmetry. One should fix, then, this symmetry by fixing position of three vertices in the real line. Different fixing of these positions give different ordering of the four vertices in the boundary of the world-sheet. One should add all non-cyclic permutation of the vertices to get the correct scattering amplitude. So one should add the amplitudes resulting from the fixing $(x_1 = 0, x_2, x_3 = 1, x_4 = \infty)$, $(x_1 = 0, x_2, x_4 = 1, x_3 = \infty)$, $(x_1 = 0, x_3, x_4 = 1, x_2 = \infty)$, $(x_1 = 0, x_3, x_2 = 1, x_4 = \infty)$, $(x_1 = 0, x_4, x_2 = 1, x_3 = \infty)$, $(x_1 = 0, x_4, x_3 = 1, x_2 = \infty)$. After these gauge fixing, one ends up with only one integral which gives the beta function. The S-matrix elements that we will analyze in this paper can be found in the standard books [44, 45]. The S-matrix elements in the discussion section which are not in [44, 45], can easily be performed using the above prescription.

2.1 Four tachyons amplitude

The S-matrix element of four open string tachyon vertex operators in the supersting theory is given by [44, 45]

$$A^{\rm tachyon} \sim \alpha\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} + \beta\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + \gamma\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)}\,, \tag{2.1}$$

where the Mandelstam variables are

$$s = -\frac{\alpha'(k_1+k_2)^2}{2}\,,$$
$$t = -\frac{\alpha'(k_2+k_3)^2}{2}\,,$$
$$u = -\frac{\alpha'(k_1+k_3)^2}{2}\,. \tag{2.2}$$

The coefficients α, β, γ are the non-abelian group factors

$$\alpha = \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_4) + \mathrm{Tr}(\lambda_1\lambda_4\lambda_3\lambda_2)\right),$$
$$\beta = \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_3\lambda_4\lambda_2) + \mathrm{Tr}(\lambda_1\lambda_2\lambda_4\lambda_3)\right),$$
$$\gamma = \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_4\lambda_2\lambda_3) + \mathrm{Tr}(\lambda_1\lambda_3\lambda_2\lambda_4)\right). \tag{2.3}$$

The on-shell condition for the tachyons are $k_i^2 = 1/(2\alpha')$, and the Mandelstam variables satisfy the constraint

$$s + t + u = -1\,. \tag{2.4}$$

Standard non-abelian kinetic term in field theory produces massless poles in s-, t-, u-channels. However, the constraint (2.4) does not allow us to sent all s, t, u to zero at the same time, i.e., $s, t, u \to 0$, to produce massless poles. In the non-commutative case, it is shown in [55] that in order to produce the massless poles and contact terms resulting from non-commutative kinetic term, one should arrange the amplitude in a specific forms, i.e., $A = A_s + A_t + A_u$, and then send $s \to 0$, $t, u \to -1/2$ in A_s to produce massless pole in s-channel, $t \to 0$, $s, u \to -1/2$ in A_t to produce massless pole in t-channel, and send $u \to 0$, $s, t \to -1/2$ in A_u to produce massless pole in the u-channel. Similar thing happens here. That is, one should write $A^{\rm tachyon} = A_s^{\rm tachyon} + A_t^{\rm tachyon} + A_u^{\rm tachyon}$ where

$$\begin{aligned}
A_s^{\rm tachyon} &\sim \alpha\frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)} + \beta\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} - \\
&\quad - \gamma\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)}\,, \\
A_u^{\rm tachyon} &\sim -\alpha\frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)} + \beta\frac{\Gamma(-2u)\Gamma(-2s)}{\Gamma(-1-2u-2s)} + \\
&\quad + \gamma\frac{\Gamma(-2u)\Gamma(-2t)}{\Gamma(-1-2u-2t)}\,, \\
A_t^{\rm tachyon} &\sim \alpha\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} - \beta\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + \\
&\quad + \gamma\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)}\,.
\end{aligned} \tag{2.5}$$

The field theory massless poles and contact terms resulting from non-abelian kinetic term are reproduced by sending

$$\begin{aligned}
s-\text{channel}: &\quad \lim_{s\to 0\,,t,u\to -1/2} A_s^{\text{tachyon}}\\
t-\text{channel}: &\quad \lim_{t\to 0\,,s,u\to -1/2} A_t^{\text{tachyon}}\\
u-\text{channel}: &\quad \lim_{u\to 0\,,s,t\to -1/2} A_s^{\text{tachyon}}\,.
\end{aligned} \tag{2.6}$$

It may seems strange that in this limit one should send s, say, once to zero and once to $-1/2$. However, this happens only in the particular form of the amplitude (2.5). We will show now that one can use the constraint (2.4) to rewrite the amplitude (2.5) in such a way that the limit corresponds to sending all s,t,u to zero. Consider the gamma functions in the first term in A_t^{tachyon}. In the above limit, it has the following expansion:

$$\begin{aligned}
\lim_{t\to 0;s,u\to -1/2}\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2s-2t)} &= \frac{1+2u}{-2t}+\frac{2\pi^2}{3}\left(u+\frac{1}{2}\right)\left(s+\frac{1}{2}\right)+\cdots\\
&= \frac{1}{2}+\frac{u-s}{-2t}+\frac{\pi^2}{6}(u-s-t)(s-t-u)+\cdots\\
&= \lim_{s,t,u\to 0}\frac{\Gamma(-2t)\Gamma(1-s+t+u)}{\Gamma(u-t-s)}\,,
\end{aligned}$$

where in the second line above we have used the constraint (2.4). In the third line we write the gamma functions that produce terms in the second line at the limit $s,t,u\to 0$. Similar thing can be done for all other gamma functions in (2.5). The result is

$$\begin{aligned}
A_s^{\text{universal}} &\sim \alpha\frac{\Gamma(-2s)\Gamma(1+s+u-t)}{\Gamma(u-s-t)}+\beta\frac{\Gamma(-2s)\Gamma(1+s+t-u)}{\Gamma(t-s-u)}-\\
&\quad -\gamma\frac{\Gamma(1+s+u-t)\Gamma(1+s+t-u)}{\Gamma(1+2s)}\,,\\
A_u^{\text{universal}} &\sim -\alpha\frac{\Gamma(1+t+u-s)\Gamma(1+s+u-t)}{\Gamma(1+2u)}+\beta\frac{\Gamma(-2u)\Gamma(1+u+t-s)}{\Gamma(t-u-s)}+\\
&\quad +\gamma\frac{\Gamma(-2u)\Gamma(1+s+u-t)}{\Gamma(s-u-t)}\,,\\
A_t^{\text{universal}} &\sim \alpha\frac{\Gamma(-2t)\Gamma(1+t+u-s)}{\Gamma(u-t-s)}-\beta\frac{\Gamma(1+t+u-s)\Gamma(1+s+t-u)}{\Gamma(1+2t)}+\\
&\quad +\gamma\frac{\Gamma(-2t)\Gamma(1+s+t-u)}{\Gamma(s-t-u)}\,.
\end{aligned} \tag{2.7}$$

Now the non-abelian limit is $s,t,u\to 0$. We have changed the superscript A^{tachyon} to $A^{\text{universal}}$. The reason is that the S-matrix element of massless scalars can also be written in the above form. To see this consider the S-matrix element of four scalar vertex operators [44, 45], that is[3] $A^{\rm scalar}=A_s^{\rm scalar}+A_t^{\rm scalar}+A_u^{\rm scalar}$ where

$$\begin{aligned}
A_s^{\rm scalar} &\sim \zeta_1\!\cdot\!\zeta_2\zeta_3\!\cdot\!\zeta_4\bigg(\alpha\frac{\Gamma(-2s)\Gamma(1-2t)}{\Gamma(-2s-2t)}+\beta\frac{\Gamma(-2s)\Gamma(1-2u)}{\Gamma(-2s-2u)}-\\
&\qquad -\gamma\frac{\Gamma(1-2t)\Gamma(1-2u)}{\Gamma(1-2t-2u)}\bigg),\\
A_u^{\rm scalar} &\sim \zeta_1\!\cdot\!\zeta_3\zeta_2\!\cdot\!\zeta_4\bigg(-\alpha\frac{\Gamma(1-2s)\Gamma(1-2t)}{\Gamma(1-2s-2t)}+\beta\frac{\Gamma(-2u)\Gamma(1-2s)}{\Gamma(-2u-2s)}+\\
&\qquad +\gamma\frac{\Gamma(-2u)\Gamma(1-2t)}{\Gamma(-2u-2t)}\bigg),\\
A_t^{\rm scalar} &\sim \zeta_1\!\cdot\!\zeta_4\zeta_2\!\cdot\!\zeta_3\bigg(\alpha\frac{\Gamma(-2t)\Gamma(1-2s)}{\Gamma(-2t-2s)}-\beta\frac{\Gamma(1-2s)\Gamma(1-2u)}{\Gamma(1-2s-2u)}+\\
&\qquad +\gamma\frac{\Gamma(-2t)\Gamma(1-2u)}{\Gamma(-2t-2u)}\bigg),
\end{aligned} \tag{2.8}$$

where ζ's are the scalars polarization, the on-shell condition for the scalars are $k_i^2=0$, and the Mandelstam variables constrain to the relation

$$s+t+u=0\,. \tag{2.9}$$

In this case the non-abelian limit is $s,t,u\to 0$. Now it is easy to check that using the constraint (2.9), the amplitude can be written in the universal form (2.7) too. We interpret the observation that both tachyon and massless scalars have identical amplitude, as an indication that the on-shell mass of tachyon does not appear in the amplitude in the universal form. It does, however, appear in non-universal form (2.5).

The non-abelian limit of the tachyon amplitude in the universal form (2.7) is $s,t,u\to 0$. In this limit, one finds the following massless pole and tower of contact terms:

$$\begin{aligned}
A^{\rm universal} &= -4iT_p\bigg(\frac{(\alpha-\beta)(t-u)}{2s}+\frac{(\beta-\gamma)(s-t)}{2u}+\frac{(\alpha-\gamma)(s-u)}{2t}+\\
&\qquad +\zeta(2)(\alpha+\beta+\gamma)\left(2ut+2st+2su-u^2-t^2-s^2\right)+\cdots\bigg),
\end{aligned} \tag{2.10}$$

where we have also normalized the amplitude by the factor of $-4T_pi$. The Zeta function is $\zeta(2)=\pi^2/6$, and dots represents terms of order cubic and more in the Mandelstam variables. These terms are ordered in terms of higher Zeta functions, i.e., $\zeta(3),\zeta(4),\ldots$.

Now in field theory we calculate the same S-matrix element. Using the free action, one finds that $k_i^2=-m^2$, and the Mandelstam variables become

$$\begin{aligned}
s &= -\frac{\alpha'(k_1+k_2)^2}{2}=-\frac{\alpha'(-2m^2+2k_1\!\cdot\!k_2)}{2},\\
t &= -\frac{\alpha'(k_2+k_3)^2}{2}=-\frac{\alpha'(-2m^2+2k_2\!\cdot\!k_3)}{2},\\
u &= -\frac{\alpha'(k_1+k_3)^2}{2}=-\frac{\alpha'(-2m^2+2k_1\!\cdot\!k_3)}{2}\,.
\end{aligned} \tag{2.11}$$

[3] The S-matrix element of the scalar vertex operators can be read from the S-matrix element of gauge field vertex operators by restricting the polarization of the gauge fields to transverse directions and the momentum of the gauge fields to the world-volume directions.

JHEP12(2003)036

The on-shell condition constrains the Mandelstam variables in the relation

$$s+t+u=2\alpha' m^2\,. \tag{2.12}$$

Since tachyon mass does not appear in (2.10), the mass m above can be arbitrary. Using the Feynman rules for evaluating S-matrix elements in field theory, one realizes that the massless poles in (2.10) are reproduced by non-abelian kinetic term, and the contact terms of order $\zeta(2)$ are reproduced by the gauge invariant couplings in the second line below:[4]

$$\begin{aligned}
&-T_p\mathrm{Tr}\left(\frac{(2\pi\alpha')}{2}D_aTD^aT-\frac{(2\pi\alpha')^2}{4}F_{ab}F^{ba}\right)-\\
&-(2\pi\alpha')^2T_p\mathrm{STr}\left(\frac{m^4}{8}T^4+\frac{m^2}{4}T^2D_aTD^aT-\frac{1}{4}(D_aTD^aT)^2\right),
\end{aligned} \tag{2.13}$$

where the covariant derivative is $D_aT=\partial_aT-i[A_a,T]$, and STr is the symmetrised trace prescription. In reaching to this result we have used several times the kinematic relations (2.11) and (2.12). Note that the mass m appears as coefficient of different terms. All the terms in the second line above are of the same order of α'. The higher order terms in (2.10) which have coefficient of higher Zeta function i.e., $\zeta(n)$ with $n>2$ are related to the higher covariant derivatives of T and higher power of m in which we are not interested in the present paper.

2.2 Two tachyons and two gauge fields amplitude

The S-matrix element of two gauge and two tachyon vertex operators is given by [44, 45]

$$\begin{aligned}
A^{\mathrm{tachyon}}\sim\ &\frac{1}{2}\zeta_1\!\cdot\!\zeta_2\biggl(-\alpha\frac{\Gamma(-2s)\Gamma(1/2-2t)}{\Gamma(-1/2-2s-2t)}-\beta\frac{\Gamma(-2s)\Gamma(1/2-2u)}{\Gamma(-1/2-2s-2u)}+\\
&+\gamma\frac{\Gamma(1/2-2u)\Gamma(1/2-2t)}{\Gamma(1+2s)}\biggr)+\\
&+2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4\biggl(\alpha\frac{\Gamma(-2s)\Gamma(1/2-2t)}{\Gamma(1/2-2s-2t)}-\beta\frac{\Gamma(-2s)\Gamma(-1/2-2u)}{\Gamma(-1/2-2s-2u)}+\\
&+\gamma\frac{\Gamma(-1/2-2u)\Gamma(1/2-2t)}{\Gamma(-2t-2u)}\biggr)+3\leftrightarrow 4\,.
\end{aligned} \tag{2.14}$$

The Mandelstam variables satisfy the on-shell constraint

$$s+t+u=-\frac{1}{2}\,. \tag{2.15}$$

The non-abelian kinetic terms of gauge field and tachyon produce Feynman amplitudes that have massless pole in only s-channel, tachyonic poles in t- and u-channels, and some contact terms. So, in the amplitude (2.14), one should send $s\to 0$ and $t,u\to -1/4$. This limit is consistent with the constraint (2.15). Hence, it is the non-abelian limit of the amplitude (2.14). Here again one may use the constraint (2.15) to change the limit to

[4] We didn't try gauge invariant couplings like $T^2DDTDDT$. We expect these terms appear in the next order which includes second covariant derivative terms.

$s, t, u \to 0$, e.g.,

$$\lim_{s\to 0;t,u\to -1/4}\frac{\Gamma(-2s)\Gamma(1/2-2t)}{-1/2-2t-2s} = \frac{1/2+2u}{-2s}+\frac{2\pi^2}{3}\left(u+\frac{1}{4}\right)\left(t+\frac{1}{4}\right)+\cdots$$
$$= \frac{1}{2}+\frac{u-t}{-2s}+\frac{\pi^2}{6}(u-s-t)(t-s-u)+\cdots$$
$$= \lim_{s,t,u\to 0}\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(u-t-s)}\,,$$

where we have used the constraint (2.15) in the second line above. Similarly for all other gamma functions in (2.14). The amplitude in the universal form is then

$$\begin{aligned}
A^{\rm universal} \sim\ & \frac{1}{2}\zeta_1\!\cdot\!\zeta_2\Bigg(-\alpha\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(u-s-t)}-\beta\frac{\Gamma(-2s)\Gamma(1-u+t+s)}{\Gamma(t-s-u)}+\\
&+\gamma\frac{\Gamma(1-u+s+t)\Gamma(1-t+u+s)}{\Gamma(1+2s)}\Bigg)+\\
&+2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4\Bigg(\alpha\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(1+u-s-t)}-\beta\frac{\Gamma(-2s)\Gamma(-u+t+s)}{\Gamma(t-s-u)}+\\
&+\gamma\frac{\Gamma(-u+t+s)\Gamma(1-t+u+s)}{\Gamma(1+2s)}\Bigg)+3\leftrightarrow 4\,.
\end{aligned} \tag{2.16}$$

Again the reason that we have called the above amplitude as universal amplitude is that the S-matrix element of two gauge fields and two scalars can also be written in this form. To see this consider this amplitude in the standard form [44, 45]

$$\begin{aligned}
A^{\rm scalar} \sim\ & \zeta_3\!\cdot\!\zeta_4\Bigg\{\frac{1}{2}\zeta_1\!\cdot\!\zeta_2\Bigg(-\alpha\frac{\Gamma(-2s)\Gamma(1-2t)}{\Gamma(-2s-2t)}-\beta\frac{\Gamma(-2s)\Gamma(1-2u)}{\Gamma(-2s-2u)}+\\
&+\gamma\frac{\Gamma(1-2u)\Gamma(1-2t)}{\Gamma(1+2s)}\Bigg)+\\
&+2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4\Bigg(\alpha\frac{\Gamma(-2s)\Gamma(1-2t)}{\Gamma(1-2s-2t)}-\beta\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-2s-2u)}+\\
&+\gamma\frac{\Gamma(-2u)\Gamma(1-2t)}{\Gamma(1-2t-2u)}\Bigg)+3\leftrightarrow 4\Bigg\}\,.
\end{aligned}$$

In this case the Mandelstam variables satisfy the constraint (2.9). Now it is not difficult to check that, using the constraint (2.9), the gamma functions in above amplitude can be written in the universal form (2.16).

Writing the amplitude in the universal form (2.16), one finds its non-abelian expansion by sending the Mandelstam variables in it to zero, i.e., $s, t, u \to 0$. We are not interested in the contact terms which have the Zeta function $\zeta(3)$ and more, since they are of higher order of α'. So in the first big parentheses of (2.16) we should keep the following terms of the gamma expansion:

$$\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(u-t-s)} = \frac{1}{2}+\frac{t-u}{2s}+\zeta(2)(s^2-(u-t)^2)+\cdots\,,$$
$$\frac{\Gamma(-2s)\Gamma(1-u+t+s)}{\Gamma(t-u-s)} = \frac{1}{2}+\frac{u-t}{2s}+\zeta(2)(s^2-(u-t)^2)+\cdots\,,$$
$$\frac{\Gamma(1-u+s+t)\Gamma(1-t+u+s)}{\Gamma(1+2s)} = 1-\zeta(2)(s^2-(u-t)^2)+\cdots\,,$$

and in the second parentheses we should keep the following terms:

$$\begin{aligned}
\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(1+u-t-s)} &= -\frac{1}{2s} - \zeta(2)(-t+u+s) + \cdots\,, \\
\frac{\Gamma(-2s)\Gamma(-u+t+s)}{\Gamma(t-u-s)} &= -\frac{1}{2s} + \frac{1}{-u+s+t} + \zeta(2)(-t+u+s) + \cdots\,, \\
\frac{\Gamma(-u+t+s)\Gamma(1-t+u+s)}{\Gamma(1+2s)} &= \frac{1}{-u+s+t} - \zeta(2)(-t+u+s) + \cdots\,.
\end{aligned}$$

Replacing the above expansion for the gamma function in (2.16), one finds the following leading terms:

$$\begin{aligned}
A^{\text{universal}} &= 4i(2\pi\alpha')T_p\Bigg\{\frac{1}{2}\zeta_1\!\cdot\!\zeta_2\left(-\frac{\alpha+\beta}{2}+\gamma\right) - \frac{\alpha-\beta}{2s}\times \\
&\qquad \times\left(\frac{1}{2}(t-u)\zeta_1\!\cdot\!\zeta_2 + 2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4\right) - \\
&\qquad - \frac{\gamma-\beta}{u-t-s}\left(2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4\right) - \zeta(2)(\alpha+\beta+\gamma)\times \\
&\qquad \times\left(\frac{1}{2}\zeta_1\!\cdot\!\zeta_2(s^2-(u-t)^2) + 2\alpha'\zeta_1\!\cdot\! k_3\zeta_2\!\cdot\! k_4(-t+u+s)\right) + \cdots\Bigg\}
\end{aligned} \tag{2.17}$$

plus terms with $3 \leftrightarrow 4$. We have normalized the amplitude at this point by the factor $4i(2\pi\alpha')T_p$. In above equation, dots represents contact terms that have $\zeta(3)$ or higher Zeta functions. Note that the tachyon mass does not appear in the universal amplitude (2.16), nor in its above expansion.

Now in field theory, using the fact that particle 1, 2 are massless gauge fields, and 3, 4 are tachyon with mass m, the Mandelstam variables become:

$$\begin{aligned}
s &= -\frac{\alpha'(k_1+k_2)^2}{2} = -\frac{\alpha'(2k_1\!\cdot\! k_2)}{2}\,, \\
t &= -\frac{\alpha'(k_2+k_3)^2}{2} = -\frac{\alpha'(-m^2+2k_2\!\cdot\! k_3)}{2}\,, \\
u &= -\frac{\alpha'(k_1+k_3)^2}{2} = -\frac{\alpha'(-m^2+2k_1\!\cdot\! k_3)}{2}\,.
\end{aligned} \tag{2.18}$$

Conservation of momentum constrains them in the relation

$$s+t+u = \alpha' m^2\,. \tag{2.19}$$

Consider adding the following terms to the action (2.13):

$$\begin{aligned}
&-T_p\mathrm{STr}\Bigg(\pi\alpha' m^2T^2 + (2\pi\alpha')^3\times \\
&\qquad\times\left(-\frac{1}{8}m^2T^2F_{ab}F^{ba} + \frac{1}{2}F_{ab}F^{bc}D_cTD^aT - \frac{1}{8}D_aTD^aTF_{bc}F^{cb}\right)\Bigg)\,.
\end{aligned}$$

It is a simple exercise to verify that the commutator $[A_a,T][A^a,T]$ of the tachyon kinetic term reproduces the terms in the first line of (2.17). The Feynman diagram with one vertex $\partial_aT[A^a,T]$ from the tachyon kinetic term, the other vertex $\partial_aA_b[A^a,A^b]$ from the kinetic term of the gauge field, and the gauge field as propagator reproduces the massless pole in (2.17). The diagram with the two vertices $\partial_a T[A^a, T]$ from the tachyon kinetic term and tachyon as propagator, i.e., $G_T = i/(\pi\alpha' T_p(2u - \alpha' m^2)) = i/(\pi\alpha' T_p(u - s - t))$, reproduces the tachyonic pole in (2.17). All these confirm that we expanded the amplitude correctly, i.e., it produces the Feynman amplitudes resulting from non-abelian kinetic term. It is easy to verify that the next leading terms in the third line of (2.17) are reproduce by the contact terms above.

The action (2.13) and above couplings are the leading terms of the expansion of the following symmetrised trace non-abelian tachyonic BI action

$$\mathcal{L}^{\rm BI} = -T_p \,\mathrm{STr}\left(V(T)\sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a T D_b T)}\right), \tag{2.20}$$

where the tachyon potential has the expansion i.e.,

$$V(T) = 1 + \pi\alpha' m^2 T^2 + \frac{1}{2}(\pi\alpha' m^2 T^2)^2 + \cdots . \tag{2.21}$$

Now it raises the question that if one analyzes the higher S-matrix elements, would one find consistency between the leading terms of the S-matrix elements and the above action? The above action in the abelian case when tachyon freezes at $T = 0$, reduces to the Born-Infeld action [53, 47].[5] In that case, the non-abelian extension of the action is proposed to be the symmetrised trace of non-abelian generalization of Born-Infeld action [50]. The leading terms of S-matrix element of four gauge fields at the low-energy/non-abelian limit was shown to be consistent with this non-abelian action [51]. However, there are indication that the symmetrised trace prescription does not work for F^6 [52]. Using the fact that the massless scalar couplings can be added to the Born-Infeld action using the T-duality rules [46], one expects that the coupling of six scalars not to be consistent with the symmetrised trace prescription either. Now, extending the observation made here for four-point function that the S-matrix element of tachyon and scalar vertex operators can be written in the universal form, to the six-point function, one may conclude that the symmetrised trace prescription in (2.20) does not work for the S-matrix elements of six tachyons either. However, restricting the results to abelian case, one may expect to have consistency with abelian tachyonic BI action.

3. Amplitudes in bosonic string theory

In this section we will analyze the S-matrix element of four tachyons, and the S-matrix element of two tachyons and two gauge fields in the bosonic theory.

3.1 Four tachyons amplitude

The S-matrix element of four tachyon vertex operators in the bosonic string theory is given by [44, 45]

$$A^{\rm tachyon} \sim -3\left(\alpha B(-2s-1, -2t-1) + \beta B(-2s-1, -2u-1) + \gamma B(-2u-1, -2t-1)\right).$$

[5]It should be emphasized here that the BI action is an action to all order of α'. The effective action to a fixed order of α', however, has many other terms which has less power of field strength but higher derivative of it. Only in the situations that the higher derivative terms are zero, the BI action is an effective action.

The on-shell condition for tachyon momentum is $k^2 = 1/\alpha'$, and the Mandelstam variables satisfy the constraint

$$s + t + u = -2\,. \tag{3.1}$$

The extra factor of -3 in the amplitude is for later use.

To find the non-abelian limit of this amplitude, i.e., the limit that produces Feynman amplitudes resulting from non-abelian kinetic term, one may compare above amplitude with the S-matrix element of four scalars. This because we know how to find the non-abelian limit in that case. This trick was also used in [55] to find the limit (2.6) in superstring case.

The S-matrix element of four scalar vertex operators in the bosonic theory is given as [44, 45], $A^{\rm scalar} = A_s^{\rm scalar} + A_t^{\rm scalar} + A_u^{\rm scalar}$ where

$$\begin{aligned}
A_s^{\rm scalar} &\sim \zeta_1\!\cdot\!\zeta_2\zeta_3\!\cdot\!\zeta_4\Big(\alpha B(-2s-1,-2t+1)+\beta B(-2s-1,-2u+1)+\\
&\qquad\qquad +\gamma B(-2u+1,-2t+1)\Big)\\
A_u^{\rm scalar} &\sim \zeta_1\!\cdot\!\zeta_3\zeta_2\!\cdot\!\zeta_4\Big(\alpha B(-2s+1,-2t+1)+\beta B(-2s+1,-2u-1)+\\
&\qquad\qquad +\gamma B(-2u-1,-2t+1)\Big)\\
A_t^{\rm scalar} &= \zeta_1\!\cdot\!\zeta_4\zeta_2\!\cdot\!\zeta_3\Big(\alpha B(-2s+1,-2t-1)+\beta B(-2s+1,-2u+1)+\\
&\qquad\qquad +\gamma B(-2u+1,-2t-1)\Big)\,.
\end{aligned} \tag{3.2}$$

Note that $A_s^{\rm scalar}$, $A_t^{\rm scalar}$, and $A_u^{\rm scalar}$ have tachyonic and massless poles only in s-, t-, and u-channel, respectively. In this case, we know that the Feyman amplitudes resulting from non-abelian kinetic term are reproduced by above amplitude when expanding it at $s,t,u\rightarrow 0$. Moreover, all the Beta functions have contribution to these amplitudes. A minute thinking about these facts, one realizes that to find the non-abelian limit of the tachyon amplitude, one should write it as $A^{\rm tachyon} = A_s^{\rm tachyon} + A_t^{\rm tachyon} + A_u^{\rm tachyon}$ where $A_s^{\rm tachyon} = A_t^{\rm tachyon} = A_u^{\rm tachyon}$ and

$$A_s^{\rm tachyon} = -\left(\alpha B(-2s-1,-2t-1)+\beta B(-2s-1,-2u-1)+\gamma B(-2u-1,-2t-1)\right)$$

The amplitude $A_s^{\rm tachyon}$, $A_s^{\rm tachyon}$, and $A_s^{\rm tachyon}$, should produce the massless pole in s-, t- and u-channel, respectively, that the non-abelian tachyon kinetic produces. The non-abelian limit is then

$$\begin{aligned}
s-\text{channel}:&\quad \lim_{s\rightarrow 0\,,t,u\rightarrow -1} A_s^{\rm tachyon}\\
t-\text{channel}:&\quad \lim_{t\rightarrow 0\,,s,u\rightarrow -1} A_t^{\rm tachyon}\\
u-\text{channel}:&\quad \lim_{u\rightarrow 0\,,s,t\rightarrow -1} A_s^{\rm tachyon}\,.
\end{aligned} \tag{3.3}$$

Note that in this limit, for instance, the tachyonic pole of $A_s^{\rm tachyon}$ in s-channel and massless and tachyonic poles in t and u channels must be expanded. Similarly for $A_t^{\rm tachyon}$ and $A_u^{\rm tachyon}$. Again the above limit can be changed to the $s,t,u\to 0$ limit by imposing the constraint (3.1) to rewrite the amplitude in the universal form. The amplitude, in the universal form, in this case has extra tachyonic pole relative to the amplitude in the superstring case [57], i.e.,

$$\begin{aligned}
A_s^{\rm bosonic\,string} &= \frac{1}{1+2s}A_s^{\rm superstring}\,,\\
A_u^{\rm bosonic\,string} &= \frac{1}{1+2u}A_u^{\rm superstring}\,,\\
A_t^{\rm bosonic\,string} &= \frac{1}{1+2t}A_t^{\rm superstring}\,,
\end{aligned} \tag{3.4}$$

where the $A^{\rm superstring}$,s are given in (2.7). Using the constraint (2.9), it is easy to check that the scalar amplitude (3.2) can be rewritten in the above universal form too. Note that the amplitudes $A_s\,, A_t$, and A_u are not identical in the universal from. Writing $1/(1+2x) = -2x/(1+2x)+1$, one may rewrite the universal amplitude as $A^{\rm universal} = A^{\rm superstring}+A^{\rm extra}$ where

$$\begin{aligned}
A_s^{\rm extra} \sim \frac{1}{1+2s}\bigg(&\alpha\frac{\Gamma(1-2s)\Gamma(1+s+u-t)}{\Gamma(u-s-t)}+\beta\frac{\Gamma(1-2s)\Gamma(1+s+t-u)}{\Gamma(t-s-u)}+\\
&+\gamma\frac{\Gamma(1+s+t-u)\Gamma(1+s+u-t)}{\Gamma(2s)}\bigg)\,,
\end{aligned}$$

similar expressions for $A_u^{\rm extra}$ and $A_t^{\rm extra}$.

The non-abelian limit of the amplitude in the universal form is $s,t,u\to 0$. Expansion of the $A^{\rm superstring}$ part at this limit has been already discussed in previous section. The $A^{\rm extra}$ part has no massless pole but has tachyonic poles. These tachyonic poles are not related to the Feynman amplitudes in field theory that result from kinetic term. They are related to the three tachyon couplings in field theory. However, in the non-abelian limit, i.e., $s,t,u\to 0$, these poles must be expanded. This is what has been done in [53, 47, 54, 58] to find the gauge invariant/covariant couplings of massless fields. Since these tachyonic poles must be expanded, they indicate that the non-abelian field theory may have no three tachyon coupling. In fact, expansion of $A^{\rm extra}$ at the non-abelian limit is

$$\begin{aligned}
A^{\rm extra} &= -4iT_p\left(2(\alpha-\beta)(u-t)-2s(\alpha-\beta)(u-t)-4\gamma s^2+2s^2(\alpha+\beta)\right)-\\
&\quad-4iT_p\left(2(\gamma-\alpha)(s-u)-2t(\gamma-\alpha)(s-u)-4\beta t^2+2t^2(\alpha+\gamma)\right)-\\
&\quad-4iT_p\left(2(\beta-\gamma)(t-s)-2u(\beta-\gamma)(t-s)-4\alpha u^2+2u^2(\beta+\gamma)\right)+\cdots\,,
\end{aligned} \tag{3.5}$$

where dots represents contact terms which include Mandelstam variables in cubic form or more. Note that the above leading terms are zero in the abelian case. Now consider adding the following non-abelian gauge invariant couplings to the action (2.20):

$$\begin{aligned}
\mathcal{L}_1^{\rm extra} = (2\alpha')^2T_p{\rm Tr}\Big\{&\pi iF^{ab}D_aTD_bT+m^2TD_aTTD^aT-m^2TD_aTD^aTT-\\
&-\frac{1}{2}D_aTD^aTD_bTD^bT+\frac{1}{2}D_aTD_bTD^aTD^bT\Big\}.
\end{aligned} \tag{3.6}$$

The tachyon kinetic term and the first term above give the vertex function for two external tachyons and one internal gauge field. The propagator for internal gauge field can also be read from the gauge field kinetic term. They are

$$\begin{aligned} V^a_{ij}(T_1T_2) &= (2\pi\alpha')iT_p\left((\lambda_1\lambda_2)_{ij}-(\lambda_2\lambda_1)_{ij}\right)(1-2s)(k^a_1-k^a_2)\,, \\ (G_A)^{ab}_{ij,kl} &= \frac{\delta^{ab}\delta_{jk}\delta_{il}}{(2\pi\alpha')^2T_p}\frac{i}{s}\,. \end{aligned} \tag{3.7}$$

Now the s-channel Feynman diagram $V(T_1T_2)G_AV(T_3T_4)$ produces the massless pole in the first line of (2.10) and the first two terms in first line of (3.5). Similarly for the t-channel and u-channel. The other contact terms in (3.5) are reproduced by the four tachyons couplings in (3.6). It is important to note that the above action does not have three tachyon coupling. The fact that the non-abelian action in the bosonic case has the couplings (3.6), indicates that the symmetrised trace prescription does not work in the bosonic case even for four point function.

3.2 Two tachyons and two gauge fields amplitude

The S-matrix element of two tachyons and two gauge fields in the bosonic theory is given by [44, 45]

$$\begin{aligned} A^{\rm tachyon} \sim& \left(-\frac{1}{2}\zeta_1\!\cdot\!\zeta_2+2\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_3\right)\times \\ &\times\left(\alpha B(-1-2s,-2t)+\beta B(-1-2s,-2u)+\gamma B(-2u,-2t)\right)+2\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_4\times \\ &\times\left(\alpha B(-1-2s,1-2t)+\beta B(-1-2s,-1-2u)-\gamma B(-1-2u,1-2t)\right)+3\leftrightarrow 4\,. \end{aligned} \tag{3.8}$$

The Mandelstam variables satisfy

$$s+t+u=-1\,. \tag{3.9}$$

As can be seen there are massless and tachyonic poles in all s, u and t channels. A non-trivial question is: which of these should be expanded and which ones should be reproduced by the non-abelian gauge theory? To answer this question, one has to find the limit that reduces the above amplitude to a series that has tachyonic poles, massless poles and contact terms which are reproduced by the non-abelian gauge theory. Since the kinetic term in the gauge theory is identical in both superstring theory and bosonic theory, the non-abelian limit in bosonic amplitude is similar to the non-abelian limit in the superstring case. However, the constraint here (3.9) is different than the constraint there (2.15). The non-abelian limit in the present case is, then, $s\to 0$ and $t,u\to -1/2$. Therefore, in this limit, the tachyonic pole in the s-channel must be expended. This is consistent with the observation made in previous subsection that field theory has no three tachyons coupling, at least to the order that we consider in this paper. Similarly, in the non-abelian limit, the massless poles in the t and u channels must be expanded which indicates that field theory may have no gauge-gauge-tachyon coupling. Now, like in the superstring case, one may use the constraint (3.9) to change this limit to the limit $s, t, u \to 0$, that is,

$$
\begin{aligned}
A^{\text{universal}} \sim & \left(-\frac{1}{2}\zeta_1\cdot\zeta_2 + 2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_3\right) \times \\
& \times\Big(\alpha B(-1-2s, 1-t+u+s) + \beta B(-1-2s, 1-u+t+s) + \\
& + \gamma B(1-u+t+s, 1-t+u+s)\Big) + 2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_4 \times \\
& \times\Big(\alpha B(-1-2s, 2-t+u+s) + \beta B(-1-2s, -u+s+t) - \\
& - \gamma B(-u+t+s, 2-t+u+s)\Big) + 3 \leftrightarrow 4\,.
\end{aligned}
\tag{3.10}
$$

To compare it with the S-matrix element of two gauge and two scalar vertex operators, consider the latter amplitude [44, 45],

$$
\begin{aligned}
A^{\text{scalar}} \sim \zeta_3\cdot\zeta_4 \Bigg\{ & \left(-\frac{1}{2}\zeta_1\cdot\zeta_2 + 2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_3\right) \times \\
& \times\Big(\alpha B(-1-2s, 1-2t) + \beta(-1-2s, 1-2u) + \gamma B(1-2u, 1-2t)\Big) + \\
& + 2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_4\Big(\alpha B(-1-2s, 2-2t) + \beta B(-1-2s, -2u) - \\
& - \gamma B(-2u, 2-2t)\Big) + 3 \leftrightarrow 4\Bigg\},
\end{aligned}
$$

where the Mandelstam variables satisfy $s + t + u = 0$. Using this constraint, one can easily check that the above amplitude can be rewritten as (3.10). That is why we call the amplitude (3.10) the universal amplitude.

In the universal form, the non-abelian limit is $s, t, u \to 0$. It is easy now to see from the universal amplitude (3.10) that the s-channel tachyonic pole of amplitude in the non-universal form (3.8) has to be expanded whereas the tachyonic poles in the u and t channels should be reproduced by the non-abelian gauge theory.

To study the non-abelian limit of the amplitude, like in the previous subsection, it is convenient to separate the amplitude into two parts. One is the amplitude that has the same dependency on the momenta as in the superstring case. We call it $A^{\text{superstring}}$. The non-abelian limit of this part is consistent with the non-abelian BI action (2.20). The other part has all other terms that appear only in the bosonic theory. We call it A^{extra}. Hence,

$$
A^{\text{universal}} = A^{\text{superstring}} + A^{\text{extra}}\,,
$$

where A^{extra} has the following terms:

$$
\begin{aligned}
2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_3 & \left(\alpha\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(u-t-s)} + \beta\frac{\Gamma(-2s)\Gamma(1-u+t+s)}{\Gamma(t-u-s)} - \right. \\
& \left. - \gamma\frac{\Gamma(1-u+t+s)\Gamma(1-t+s+u)}{\Gamma(1+2s)}\right) + 2\alpha'\zeta_1\cdot k_3\zeta_2\cdot k_4 \times
\end{aligned}
$$

JHEP12(2003)036

$$\times\Bigg(\alpha(-t+u+s)\frac{\Gamma(-2s)\Gamma(1-t+u+s)}{\Gamma(1+u-t-s)}+\beta(t-u-s)\frac{\Gamma(-2s)\Gamma(-u+s+t)}{\Gamma(t-u-s)}+$$
$$+\gamma(-t+u+s)\frac{\Gamma(-u+t+s)\Gamma(1-t+u+s)}{\Gamma(1+2s)}\Bigg)+\frac{1}{1+2s}\times$$
$$\times\Bigg\{\left(-\frac{1}{2}\zeta_1\!\cdot\!\zeta_2+2\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_3\right)\Bigg(\alpha\frac{\Gamma(1-2s)\Gamma(1-t+u+s)}{\Gamma(u-t-s)}+$$
$$+\beta\frac{\Gamma(1-2s)\Gamma(1-u+t+s)}{\Gamma(t-u-s)}+$$
$$+\gamma\frac{\Gamma(1-u+t+s)\Gamma(1-t+s+u)}{\Gamma(2s)}\Bigg)+$$
$$+2\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_4\Bigg(\alpha\frac{\Gamma(1-2s)\Gamma(2-t+u+s)}{\Gamma(1+u-t-s)}+\beta\frac{\Gamma(1-2s)\Gamma(-u+s+t)}{\Gamma(-1+t-u-s)}-$$
$$-\gamma\frac{\Gamma(-u+t+s)\Gamma(2-t+u+s)}{\Gamma(2s)}\Bigg)\Bigg\}+3\leftrightarrow 4\,. \qquad (3.11)$$

Expanding the gamma functions and the tachyonic pole $1/(1+2s)$ at $s,t,u\to 0$, after some simple algebra, one finds the following leading terms:

$$\begin{aligned}A^{\rm extra} &= -4i(2\pi\alpha')T_p\Bigg\{-\frac{\alpha-\beta}{2s}\left(\alpha'\zeta_1\!\cdot\!k_2\zeta_2\!\cdot\!k_1(t-u)\right)+\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_4(\alpha-\beta)-\\ &\quad-\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_3(\alpha+\beta-2\gamma)-2\alpha'\zeta_1\!\cdot\!k_3\zeta_2\!\cdot\!k_4(\alpha-\gamma)+\\ &\quad+\frac{1}{2}\zeta_1\!\cdot\!\zeta_2\left(\alpha(u-t-s)+\beta(t-u-s)+2\gamma s\right)+\\ &\quad+\left(s\zeta_1\!\cdot\!\zeta_2+\alpha'k_1\!\cdot\!\zeta_2k_2\!\cdot\!\zeta_1\right)(\alpha-\beta)(t-u)+\\ &\quad+\left(s\zeta_1\!\cdot\!\zeta_2+\alpha'k_1\!\cdot\!\zeta_2k_2\!\cdot\!\zeta_1\right)s(\alpha+\beta-2\gamma)+3\leftrightarrow 4+\cdots\Bigg\}. \qquad (3.12)\end{aligned}$$

The above leading terms are zero for the abelian case, i.e., when $\alpha=\beta=\gamma=1$. In reaching to the above result we have used the conservation of momentum and the identities:

$$\begin{aligned}\frac{(-t+u+s)(1-t+u+s)}{(1+2s)(-u+t+s)} &= \frac{t-s-u}{1+2s}+\frac{s+u-t}{-u+t+s}\,,\\ \frac{2s(-t+u+s)}{(1+2s)(-u+t+s)} &= \frac{-2s}{1+2s}+\frac{2s}{-u+t+s}\,.\end{aligned}$$

Note that the poles like $1/(-u+s+t)$ which appear in (3.11) are disappeared in the expanded amplitude (3.12). Now consider the non-abelian tachyonic BI action (2.20), the couplings (3.6), and the following couplings

$$\begin{aligned}\mathcal{L}_2^{\rm extra} &= (2\alpha')^2\pi T_p{\rm Tr}\Bigg\{\frac{4\pi i}{3}F^{ab}F_{ac}F_b{}^c+m^2TF_{ab}TF^{ab}-m^2TF_{ab}F^{ab}T-\\ &\quad-D_aTD^aTF_{bc}F^{bc}+D_aTF_{bc}D^aTF^{bc}\Bigg\}. \qquad (3.13)\end{aligned}$$

The gauge field kinetic term and the first term above give the following vertex function for two external gauge fields and one internal gauge field:

$$\begin{aligned} V^a_{ij}(A_1A_2) = &\,(2\pi\alpha')^2 iT_p\left((\lambda_1\lambda_2)_{ij} - (\lambda_2\lambda_1)_{ij}\right)\times \\ &\times\{\zeta_1\!\cdot\!\zeta_2(k_1^a - k_2^a) + 2k_2\!\cdot\!\zeta_1\zeta_2^a - 2k_1\!\cdot\!\zeta_2\zeta_1^a - 2s\zeta_1\!\cdot\!\zeta_2(k_1^a - k_2^a) - \\ &\quad - 2k_1\!\cdot\!\zeta_2 k_2\!\cdot\!\zeta_1(k_1^a - k_2^a)\}\,. \end{aligned}$$

Now the s-channel Feynman diagram $V(A_1A_2)G_AV(T_3T_4)$, where G_A and $V(T_3T_4)$ are given in (3.7), produces the massless pole in the second line of (2.17) and the massless pole in the first line of (3.12). It also produces some contact terms. These contact terms and the contact terms $AATT$ resulting from the first term in (3.6) reproduce the contact terms in the first four lines of (3.12). The contact terms in the last line of (3.12) are reproduced by the other couplings in (3.13). On the other hand, the first term in (3.6) can produce, in general, vertex function with one gauge and one tachyon as external states and one tachyon as internal state. So with two of these vertex functions one produces a S-matrix element for two tachyons and two gauge fields. However, imposing the on-shell condition for the external gauge field, one finds that the result is zero. This is again consistent with the string theory result (3.12) which has no tachyonic pole. It is important to note that the action (3.13) has no gauge-gauge-tachyon coupling.

4. Discussion

In this paper we have proposed that there is unique expansion/limit for tachyon S-matrix elements that corresponds to non-abelian gauge symmetry. The non-trivial question is then how to find the non-abelian limit for the S-matrix elements. We have speculated that for those S-matrix elements which, in their expansion at the non-abelian limit, there are the Feynman amplitudes resulting from the non-abelian kinetic term, the limit can easily be found. The prescription for finding the non-abelian limit of these S-matrix elements has two steps: 1- Write the S-matrix elements in the universal form. 2- Expand them as the Mandelstam variables in them go to zero. We analyze in details the S-matrix element of four tachyon vertex operators, and the S-matrix element of two tachyons and two gauge fields in favor of this proposal. We then find the gauge invariant tachyon action that is consistent with the leading terms of the S-matrix element in this expansion. In the superstring theory, the non-abelian gauge invariant action is consistent with the symmetrised trace of the direct non-abelian generalization of the tachyonic BI action (2.20). In the bosonic theory, the action has some extra gauge invariant couplings that are zero in the abelian case.

In principle, finding non-abelian tachyon action from universal S-matrix elements has less ambiguity than finding abelian tachyon action from universal S-matrix elements. In the latter case, the calculation has the ambiguity that one can replace $\alpha'\partial_a\partial^aTf(T,A)$ by $\alpha'm^2Tf(T,A)$ where m^2 is arbitrary mass, whereas, in the non-abelian case the same term appears as $\alpha'D_aD^aTf(T,A)$ which can not be replaced by the tachyon mass because it represents coupling to gauge field as well. Consistency with other S-matrix element may then fix this arbitrariness. For example, one may change the coefficient of m^4T^4 in the tachyon potential (2.13), in expense of adding coupling $m^2T^3D_aD^aT$ to this action. This ambiguity, however, can be fixed by analyzing the S-matrix element of four tachyons and one gauge field vertex operator. Moreover, the ambiguity is only between the couplings that have m as their coefficient. Because those tachyon couplings that are independent of m can be read also from the scalar couplings. The latter couplings have no ambiguity. For instance, there is no ambiguity between m^4T^4 and $T^2D_aD^aTD_bD^bT$. Mass of tachyon does not appear as the coefficient of this term. So if this term appears for tachyon, it should also appear in the massless case. However, using the fact that the massless scalars can be added into the action by imposing the T-duality rules [46], such term is not consistent with the gauge symmetry. Hence, there is no such coupling for tachyon either.

In the bosonic theory, structure of terms in (3.6) and (3.13) are like the terms one finds in analyzing the S-matrix element of four gauge fields [53]

$$\mathrm{Tr}\left(\left(\frac{4i\alpha'}{3}\right)F_{ab}[F^a{}_c,F^{bc}]+2\alpha'F^{ab}F^{cd}[F_{ab},F_{cd}]\right). \tag{4.1}$$

In this present case, because of the tachyon mass, there are some other commutators, that is,

$$\begin{aligned}\mathcal{L}_1^{\rm extra}+\mathcal{L}_2^{\rm extra} = (2\alpha')^2T_p\mathrm{Tr}\Big\{&\frac{\pi i}{2}F^{ab}[D_aT,D_bT]+\frac{2\pi^2 i}{3}F^{ab}[F_{ac},F_b{}^c]+m^2TD_aT[T,D^aT]+\\ &+\pi m^2TF_{ab}[T,F^{ab}]+\frac{1}{2}D^bTD_aT[D_bT,D^aT]+\\ &+\pi F^{bc}D_aT[F_{bc},D^aT]\Big\}.\end{aligned}$$

Since the universal S-matrix element describes the scattering amplitude of four tachyons or four massless scalars, the above coupling is valid for massless scalars too. In this case the commutators that have coefficient m^2 are zero, and the the rest can easily be derived from (4.1) using T-duality.

The leading contact terms of the off-shell S-matrix element of four tachyons (2.10) is of order $(\alpha')^2$ which are reproduced by the tachyonic BI action (2.20). The non-leading terms of (2.10) that have the Zeta function $\zeta(n)$ with $n>2$ are of order $(\alpha')^n$. They are expected to be related to the higher derivative terms in field theory. In fact the terms that have coefficient $\zeta(3)$ in the expansion of (2.10) are the following:

$$\begin{aligned}A(\alpha'^3) \sim 4\zeta(3)(&\alpha(s^2u+ut^2-su^2-u^2t)+\\ &+\beta(u^2t+s^2t-ut^2-st^2)+\gamma(st^2+su^2-s^2t-s^2u)).\end{aligned}$$

These contact terms are zero in the abelian case, i.e., when $\alpha=\beta=\gamma=1$. So these terms should be reproduced by some combination of non-abelian gauge invariant coupling like $DTDDTDDTDT$ which vanishes in the abelian case. This means in field theory at order $(\alpha')^3$ there should be no term without covariant derivative, i.e., $\zeta(3)(\alpha'm^2)^3T^4$. All contact terms at this order must be reproduced by higher covariant derivative terms. The $(\alpha')^4$ contact terms of amplitude (2.10) are non-zero in the abelian case. So one can

JHEP12(2003)036

not immediately conclude that there is no term without covariant derivative. However, the coefficient of these terms is $\zeta(4)$, so it is very unlikely that there is a term like $\zeta(4)(\alpha' m^2)^4 T^4$ in field theory. If there is such a contact term, then it should be added to the coefficient of T^4 in the expansion (2.21). In that case, the tachyon potential could not be written in a closed form. On the other hand, if the higher order terms in the expansion of the string theory amplitude are reproduced only by higher covariant derivative of the tachyon field, then (2.21) is the correct expansion for the tachyon potential. In this case, using the fact that the on-shell tachyon potential should vanish at the minimum of the potential, one may expect the following form for the tachyon potential[6] :

$$V(T) = e^{\pi\alpha' m^2 T^2} . \tag{4.2}$$

Restricting to the abelian case, if the mass m appears only in the above potential, then the abelian tachyonic BI action with the above potential would be the effective action for describing the unstable D-branes when tachyon is slowly varying field. It would be interesting then to analyze the non-leading terms of the S-matrix elements to see if the mass m does not appear in them. Structure of the higher derivative tachyon couplings that have no m as their coefficient, are expected to be found also by applying T-duality rules on the higher derivative correction to the Born-Infeld action [48].

We have interpreted that the tachyon S-matrix elements in the universal from have no reference to the mass of the tachyon vertex operator. It indicates that there is nothing special about tachyon vertex operators. Hence, one may expect that the S-matrix element of any massive scalar vertex operator, in the bosonic theory, in the class,

$$V = \lambda \int dx (\zeta_i \partial^n X^i) e^{ik\cdot X} ,$$

to be written in the universal form. In above equation, $n \geq 0$, ζ_i is polarization of the scalar states, and the on-shell condition for the momentum is $k^2 = -(n-1)/\alpha'$. For example, the S-matrix element of four of these vertex operators is $A = A_s + A_u + A_t$ where

$$\begin{aligned}
A_s &\sim \zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4 \bigg(\alpha \frac{\Gamma(-1-2s)\Gamma(2n-1-2t)}{\Gamma(2n-2-2s-2t)} + \beta \frac{\Gamma(-1-2s)\Gamma(2n-1-2u)}{\Gamma(2n-2-2s-2u)} + \\
&\qquad + \gamma \frac{\Gamma(2n-1-2t)\Gamma(2n-1-2u)}{\Gamma(4n-2-2t-2u)} \bigg) , \\
A_u &\sim \zeta_1\cdot\zeta_3\zeta_2\cdot\zeta_4 \bigg(\alpha \frac{\Gamma(2n-1-2s)\Gamma(2n-1-2t)}{\Gamma(4n-2-2s-2t)} + \beta \frac{\Gamma(-1-2u)\Gamma(2n-1-2s)}{\Gamma(2n-2-2u-2s)} + \\
&\qquad + \gamma \frac{\Gamma(-1-2u)\Gamma(2n-1-2t)}{\Gamma(2n-2-2u-2t)} \bigg) , \\
A_t &\sim \zeta_1\cdot\zeta_4\zeta_2\cdot\zeta_3 \bigg(\alpha \frac{\Gamma(-1-2t)\Gamma(2n-1-2s)}{\Gamma(2n-2-2t-2s)} + \beta \frac{\Gamma(2n-1-2s)\Gamma(2n-1-2u)}{\Gamma(4n-2-2s-2u)} + \\
&\qquad + \gamma \frac{\Gamma(-1-2t)\Gamma(2n-1-2u)}{\Gamma(2n-2-2t-2u)} \bigg) ,
\end{aligned} \tag{4.3}$$

[6]For the superstring theory, the on-shell tachyon potential (4.2) is the same as the potential one finds in the partition function approach [59].

where the Mandelstam variables are those in (2.2). They satisfy the on-shell relation

$$s+t+u=2n-2\,. \tag{4.4}$$

The non-abelian limit at which the amplitude produces massless poles and contact terms resulting from the scalars kinetic term is

$$\begin{aligned}
s-\text{channel}: &\quad \lim_{s\to 0\,,t,u\to n-1} A_s\\
t-\text{channel}: &\quad \lim_{t\to 0\,,s,u\to n-1} A_t\\
u-\text{channel}: &\quad \lim_{u\to 0\,,s,t\to n-1} A_s\,.
\end{aligned}$$

Using the relation (4.4), one can easily rewrite the amplitude in the universal form, i.e.,

$$\begin{aligned}
A_s &\sim \frac{\zeta_1\!\cdot\!\zeta_2\zeta_3\!\cdot\!\zeta_4}{1+2s}\Big\{\alpha\frac{\Gamma(-2s)\Gamma(1+s+u-t)}{\Gamma(u-s-t)}+\beta\frac{\Gamma(-2s)\Gamma(1+s+t-u)}{\Gamma(t-s-u)}-\\
&\qquad -\gamma\frac{\Gamma(1+s+u-t)\Gamma(1+s+t-u)}{\Gamma(1+2s)}\Big\}\,,\\
A_u &\sim \frac{\zeta_1\!\cdot\!\zeta_3\zeta_2\!\cdot\!\zeta_4}{1+2u}\Big\{-\alpha\frac{\Gamma(1+t+u-s)\Gamma(1+s+u-t)}{\Gamma(1+2u)}+\beta\frac{\Gamma(-2u)\Gamma(1+u+t-s)}{\Gamma(t-u-s)}+\\
&\qquad +\gamma\frac{\Gamma(-2u)\Gamma(1+s+u-t)}{\Gamma(s-u-t)}\Big\}\,,\\
A_t &\sim \frac{\zeta_1\!\cdot\!\zeta_4\zeta_2\!\cdot\!\zeta_3}{1+2t}\Big\{\alpha\frac{\Gamma(-2t)\Gamma(1+t+u-s)}{\Gamma(u-t-s)}-\beta\frac{\Gamma(1+t+u-s)\Gamma(1+s+t-u)}{\Gamma(1+2t)}+\\
&\qquad +\gamma\frac{\Gamma(-2t)\Gamma(1+s+t-u)}{\Gamma(s-t-u)}\Big\}\,.
\end{aligned}$$

If one considers D_{24}-brane, then the scalar polarization is 1, and the polarization factors above can be dropped. In this case, the above S-matrix element is exactly the amplitude for four tachyons in the universal form (3.4). On the other hand, the gauge invariant action that we have found has arbitrary mass. Hence, the non-abelian action is also the action for gauge field and the above massive scalar. When the massive scalars are more than one, i.e., D_p-brane with $p<24$, the scalar field takes the index T^i. It is not difficult to insert this index in the effective action (2.20), (3.6), and (3.13). The only thing that one has to do for on-shell massive field is to replace the on-shell mass of the field into the action. In particular the potential (4.2) for massive field has no maximum, and its minimum is at zero. Whereas, the potential for tachyon has maximum at zero and minimum at infinity. Only in the latter case there is the interesting physics of tachyon condensation.

One may ask the question: Is it possible to rewrite the tachyon S-matrix element and the S-matrix element of some other massive scalar vertex operators in the universal form? To answer this question, consider, for instance, the following vertex operator:

$$V=\lambda\int dx\,(\zeta_{ij}\partial X^i\partial X^j)e^{ik\cdot X}\,,$$

where $k^2=-1/\alpha'$. The S-matrix element of four of this operator has, among other things, terms like the following:

$$A\sim \mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}(\zeta_1^T\zeta_2\zeta_4\zeta_3^T)\frac{\Gamma(1-2s)\Gamma(3-2t)}{\Gamma(4-2s-2t)}\,,$$

where the Mandelstam variables are constrained in the relation $s+t+u=2$. One can not rewrite the above S-matrix element and the S-matrix element of four tachyons in a universal form. However, the amplitude also has terms like (4.3) in which $n=2$ and $\zeta_1\cdot\zeta_2\zeta_3\cdot\zeta_4$'s are replaced by $\mathrm{Tr}(\zeta_1\zeta_2)\mathrm{Tr}(\zeta_3\zeta_4)$'s. These latter terms which produce massless poles at the non-abelian limit, can be rewritten in the universal form. Therefore, the S-matrix element has two type of terms. Those which can be written as universal form, have couplings like the tachyon couplings, and those which can not be written in the universal form, produce other gauge invariant couplings that the tachyon action does not have them. So one may conclude that the non-abelian action for any arbitrary massive scalar state includes the tachyon action as a part it.

We have seen that for those S-matrix elements that produce at the non-abelian limit massless and tachyonic poles that are related to non-abelian kinetic term, the limit can easily be found, e.g., the limit in (3.3). However, there are other S-matrix elements that have massless and tachyonic poles that are not related to non-abelian kinetic term. In this case, we don't know how to find the non-abelian limit. For example, consider the S-matrix element of three tachyons and one gauge field. This amplitude is

$$A\sim\alpha' k_2\cdot\zeta_1\left(-\hat{\alpha}B(-1-2s,-1-2t)+\hat{\beta}B(-1-2s,-2u)+\hat{\gamma}B(-2u,-1-2t)\right)+2\leftrightarrow 3\,,$$

where the Mandelstam variables are given in (2.2) and they satisfy the relation $s+t+u=-3/2$. The coefficients $\hat{\alpha},\hat{\beta}$, and $\hat{\gamma}$ are the following group factors:

$$\begin{aligned}
\hat{\alpha} &= \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_4)-\mathrm{Tr}(\lambda_1\lambda_4\lambda_3\lambda_2)\right),\\
\hat{\beta} &= \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_3\lambda_4\lambda_2)-\mathrm{Tr}(\lambda_1\lambda_2\lambda_4\lambda_3)\right),\\
\hat{\gamma} &= \frac{1}{2}\left(\mathrm{Tr}(\lambda_1\lambda_4\lambda_2\lambda_3)-\mathrm{Tr}(\lambda_1\lambda_3\lambda_2\lambda_4)\right).
\end{aligned}$$

Note that these group factors are zero in the abelian case. Hence, the above S-matrix element is zero for the abelian case. The amplitude has massless and tachyonic poles in all channels. However, neither of them are related to only non-abelian kinetic term. The massless poles could be reproduce by kinetic term and a gauge-gauge-tachyon coupling, and the tachyonic poles could be reproduced by kinetic term and a tachyon-tachyon-tachyon coupling. However, from the study of the S-matrix element of four tachyons we have learned that the non-abelian action can not have three tachyons coupling. Similarly, from the study of the S-matrix element of two tachyons and two gauge fields we have learned that the action can not have gauge-gauge-tachyon couplings. Hence, both massless and tachyonic poles in the above amplitude must be expanded in the non-abelian limit, e.g., t must not go to 0 or -1/2. The above amplitude in the non-abelian limit, then, produces only contact terms. These contact terms, however, are all zero in the abelian case. In other words, the abelian tachyon action has no gauge-tachyon-tachyon-tachyon couplings. Since the non-abelian tachyon kinetic term does not produce gauge-tachyon-tachyon-tachyon amplitude, we don't know how to find the non-abelian limit of the above amplitude. It would be interesting, then, to find the non-abelian limit of these class of S-matrix elements by other means.

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References

[1] M. Gutperle and A. Strominger, *Spacelike branes*, *J. High Energy Phys.* **04** (2002) 018 [hep-th/0202210].

[2] A. Sen, *Rolling tachyon*, *J. High Energy Phys.* **04** (2002) 048 [hep-th/0203211].

[3] A. Sen, *Tachyon matter*, *J. High Energy Phys.* **07** (2002) 065 [hep-th/0203265].

[4] A. Sen, *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[5] A. Sen, *Time evolution in open string theory*, *J. High Energy Phys.* **10** (2002) 003 [hep-th/0207105].

[6] A. Sen, *Time and tachyon*, *Int. J. Mod. Phys.* **A 18** (2003) 4869 [hep-th/0209122].

[7] P. Mukhopadhyay and A. Sen, *Decay of unstable D-branes with electric field*, *J. High Energy Phys.* **11** (2002) 047 [hep-th/0208142].

[8] A. Strominger, *Open string creation by S-branes*, hep-th/0209090.

[9] F. Larsen, A. Naqvi and S. Terashima, *Rolling tachyons and decaying branes*, *J. High Energy Phys.* **02** (2003) 039 [hep-th/0212248].

[10] M. Gutperle and A. Strominger, *Timelike boundary Liouville theory*, *Phys. Rev.* **D 67** (2003) 126002 [hep-th/0301038].

[11] A. Maloney, A. Strominger and X. Yin, *S-brane thermodynamics*, *J. High Energy Phys.* **10** (2003) 048 [hep-th/0302146].

[12] T. Okuda and S. Sugimoto, *Coupling of rolling tachyon to closed strings*, *Nucl. Phys.* **B 647** (2002) 101 [hep-th/0208196].

[13] S.J. Rey and S. Sugimoto, *Rolling tachyon with electric and magnetic fields: t-duality approach*, *Phys. Rev.* **D 67** (2003) 086008 [hep-th/0301049].

[14] S.J. Rey and S. Sugimoto, *Rolling of modulated tachyon with gauge flux and emergent fundamental string*, *Phys. Rev.* **D 68** (2003) 026003 [hep-th/0303133].

[15] N.R. Constable and F. Larsen, *The rolling tachyon as a matrix model*, *J. High Energy Phys.* **06** (2003) 017 [hep-th/0305177].

[16] B. Chen, M. Li and F.L. Lin, *Gravitational radiation of rolling tachyon*, *J. High Energy Phys.* **11** (2002) 050 [hep-th/0209222].

[17] S. Sugimoto and S. Terashima, *Tachyon matter in boundary string field theory*, *J. High Energy Phys.* **07** (2002) 025 [hep-th/0205085].

[18] J.A. Minahan, *Rolling the tachyon in super BSFT*, *J. High Energy Phys.* **07** (2002) 030 [hep-th/0205098].

[19] N. Möller and B. Zwiebach, *Dynamics with infinitely many time derivatives and rolling tachyons*, *J. High Energy Phys.* **10** (2002) 034 [hep-th/0207107].

[20] M. Fujita and H. Hata, *Time dependent solution in cubic string field theory*, *J. High Energy Phys.* **05** (2003) 043 [hep-th/0304163].

[21] I.Y. Aref'eva, L.V. Joukovskaya and A.S. Koshelev, *Time evolution in superstring field theory on non-BPS brane, I. Rolling tachyon and energy-momentum conservation*, *J. High Energy Phys.* **09** (2003) 012 [hep-th/0301137].

[22] N. Lambert, H. Liu and J. Maldacena, *Closed strings from decaying D-branes*, hep-th/0303139.

[23] D. Gaiotto, N. Itzhaki and L. Rastelli, *Closed strings as imaginary D-branes*, hep-th/0304192.

[24] J. McGreevy and H. Verlinde, *Strings from tachyons: the $c = 1$ matrix reloated*, hep-th/0304224.

[25] J. McGreevy, J. Teschner and H. Verlinde, *Classical and quantum D-branes in 2D string theory*, hep-th/0305194.

[26] I.R. Klebanov, J. Maldacena and N. Seiberg, *D-brane decay in two-dimensional string theory*, *J. High Energy Phys.* **07** (2003) 045 [hep-th/0305159].

[27] A. Sen, *Open and closed strings from unstable D-branes*, *Phys. Rev.* **D 68** (2003) 106003 [hep-th/0305011].

[28] J.L. Karczmarek, H. Liu, J. Maldacena and A. Strominger, *UV finite brane decay*, *J. High Energy Phys.* **11** (2003) 042 [hep-th/0306132].

[29] A. Sen, *Open-closed duality at tree level*, *Phys. Rev. Lett.* **91** (2003) 181601 [hep-th/0306137].

[30] M.R. Garousi, *Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action*, *Nucl. Phys.* **B 584** (2000) 284 [hep-th/0003122].

[31] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras and S. Panda, *T-duality and actions for non-BPS D-branes*, *J. High Energy Phys.* **05** (2000) 009 [hep-th/0003221];
J. Kluson, *Proposal for non-BPS D-brane action*, *Phys. Rev.* **D 62** (2000) 126003 [hep-th/0004106].

[32] D. Kutasov and V. Niarchos, *Tachyon effective actions in open string theory*, *Nucl. Phys.* **B 666** (2003) 56 [hep-th/0304045].

[33] S.H.S. Alexander, *Inflation from D- anti-D brane annihilation*, *Phys. Rev.* **D 65** (2002) 023507 [hep-th/0105032];
A. Mazumdar, S. Panda and A. Perez-Lorenzana, *Assisted inflation via tachyon condensation*, *Nucl. Phys.* **B 614** (2001) 101 [hep-ph/0107058];
M. Fairbairn and M.H.G. Tytgat, *Inflation from a tachyon fluid?*, *Phys. Lett.* **B 546** (2002) 1 [hep-th/0204070];
S. Mukohyama, *Brane cosmology driven by the rolling tachyon*, *Phys. Rev.* **D 66** (2002) 024009 [hep-th/0204084];
A. Feinstein, *Power-law inflation from the rolling tachyon*, *Phys. Rev.* **D 66** (2002) 063511 [hep-th/0204140];
T. Padmanabhan, *Accelerated expansion of the universe driven by tachyonic matter*, *Phys. Rev.* **D 66** (2002) 021301 [hep-th/0204150];
A.V. Frolov, L. Kofman and A.A. Starobinsky, *Prospects and problems of tachyon matter cosmology*, *Phys. Lett.* **B 545** (2002) 8 [hep-th/0204187];

JHEP12(2003)036

D. Choudhury, D. Ghoshal, D.P. Jatkar and S. Panda, *On the cosmological relevance of the tachyon*, *Phys. Lett.* **B 544** (2002) 231 [hep-th/0204204];
X.Z. Li, J.G. Hao and D.J. Liu, *Can quintessence be the rolling tachyon?*, *Chin. Phys. Lett.* **19** (2002) 1584 [hep-th/0204252];
G. Shiu and I. Wasserman, *Cosmological constraints on tachyon matter*, *Phys. Lett.* **B 541** (2002) 6 [hep-th/0205003];
L. Kofman and A. Linde, *Problems with tachyon inflation*, *J. High Energy Phys.* **07** (2002) 004 [hep-th/0205121];
H.B. Benaoum, *Accelerated universe from modified chaplygin gas and tachyonic fluid*, hep-th/0205140;
M. Sami, *Implementing power law inflation with rolling tachyon on the brane*, *Mod. Phys. Lett.* **A 18** (2003) 691 [hep-th/0205146];
M. Sami, P. Chingangbam and T. Qureshi, *Aspects of tachyonic inflation with exponential potential*, *Phys. Rev.* **D 66** (2002) 043530 [hep-th/0205179];
J.C. Hwang and H. Noh, *Cosmological perturbations in a generalized gravity including tachyonic condensation*, *Phys. Rev.* **D 66** (2002) 084009 [hep-th/0206100];
T. Mehen and B. Wecht, *Gauge fields and scalars in rolling tachyon backgrounds*, *J. High Energy Phys.* **02** (2003) 058 [hep-th/0206212];
K. Ohta and T. Yokono, *Gravitational approach to tachyon matter*, *Phys. Rev.* **D 66** (2002) 125009 [hep-th/0207004];
Y.S. Piao, R.G. Cai, X.M. Zhang and Y.Z. Zhang, *Assisted tachyonic inflation*, *Phys. Rev.* **D 66** (2002) 121301 [hep-ph/0207143];
G. Shiu, S.H.H. Tye and I. Wasserman, *Rolling tachyon in brane world cosmology from superstring field theory*, *Phys. Rev.* **D 67** (2003) 083517 [hep-th/0207119];
X.Z. Li, D.J. Liu and J.G. Hao, *On the tachyon inflation*, hep-th/0207146;
J.M. Cline, H. Firouzjahi and P. Martineau, *Reheating from tachyon condensation*, *J. High Energy Phys.* **11** (2002) 041 [hep-th/0207156];
A. Buchel, P. Langfelder and J. Walcher, *Does the tachyon matter?*, *Ann. Phys. (NY)* **302** (2002) 78 [hep-th/0207235];
G.N. Felder, L. Kofman and A. Starobinsky, *Caustics in tachyon matter and other Born-Infeld scalars*, *J. High Energy Phys.* **09** (2002) 026 [hep-th/0208019];
B. Wang, E. Abdalla and R.K. Su, *Dynamics and holographic discreteness of tachyonic inflation*, *Mod. Phys. Lett.* **A 18** (2003) 31 [hep-th/0208023];
S. Mukohyama, *Inhomogeneous tachyon decay, light-cone structure and D-brane network problem in tachyon cosmology*, *Phys. Rev.* **D 66** (2002) 123512 [hep-th/0208094];
M.C. Bento, O. Bertolami and A.A. Sen, *Tachyonic inflation in the braneworld scenario*, *Phys. Rev.* **D 67** (2003) 063511 [hep-th/0208124];
J.G. Hao and X.Z. Li, *Reconstructing the equation of state of tachyon*, *Phys. Rev.* **D 66** (2002) 087301 [hep-th/0209041];
C.J. Kim, H.B. Kim and Y.B. Kim, *Rolling tachyons in string cosmology*, *Phys. Lett.* **B 552** (2003) 111 [hep-th/0210101];
F. Quevedo, *Lectures on string/brane cosmology*, *Class. and Quant. Grav.* **19** (2002) 5721 [hep-th/0210292];
G. Shiu, *Tachyon dynamics and brane cosmology*, hep-th/0210313;
J.S. Bagla, H.K. Jassal and T. Padmanabhan, *Cosmology with tachyon field as dark energy*, *Phys. Rev.* **D 67** (2003) 063504 [astro-ph/0212198];
Y.S. Piao, Q.G. Huang, X.M. Zhang and Y.Z. Zhang, *Non-minimally coupled tachyon and inflation*, *Phys. Lett.* **B 570** (2003) 1 [hep-ph/0212219];

X.Z. Li and X.H. Zhai, *The tachyon inflationary models with exact mode functions*, *Phys. Rev.* **D 67** (2003) 067501 [hep-ph/0301063];
G.W. Gibbons, *Thoughts on tachyon cosmology*, *Class. and Quant. Grav.* **20** (2003) S321 [hep-th/0301117];
M. Sami, P. Chingangbam and T. Qureshi, *Cosmological aspects of rolling tachyon*, hep-th/0301140;
C.J. Kim, H.B. Kim, Y.B. Kim and O.K. Kwon, *Cosmology of rolling tachyon*, hep-th/0301142;
F. Leblond and A.W. Peet, *SD-brane gravity fields and rolling tachyons*, *J. High Energy Phys.* **04** (2003) 048 [hep-th/0303035];
K. Hashimoto, P.M. Ho, S. Nagaoka and J.E. Wang, *Time evolution via S-branes*, *Phys. Rev.* **D 68** (2003) 026007 [hep-th/0303172];
C.J. Kim, Y.B. Kim and C.O. Lee, *Tachyon kinks*, *J. High Energy Phys.* **05** (2003) 020 [hep-th/0304180];
M. Sami and N. Dadhich, *Steep inflation followed by Born-Infeld reheating*, *Phys. Lett.* **B 568** (2003) 118 [hep-th/0304187];
P. Brax, J. Mourad and D.A. Steer, *Tachyon kinks on non BPS D-branes*, *Phys. Lett.* **B 575** (2003) 115 [hep-th/0304197];
A. Buchel and J. Walcher, *Comments on supergravity description of s-branes*, *J. High Energy Phys.* **05** (2003) 069 [hep-th/0305055];
C. Kim, Y. Kim, O.K. Kwon and C.O. Lee, *Tachyon kinks on unstable Dp-branes*, *J. High Energy Phys.* **11** (2003) 034 [hep-th/0305092];
P. Singh, M. Sami and N. Dadhich, *Cosmological dynamics of phantom field*, *Phys. Rev.* **D 68** (2003) 023522 [hep-th/0305110];
Y. Demasure and R.A. Janik, *Backreaction and the rolling tachyon: an effective action point of view*, *Phys. Lett.* **B 578** (2004) 195 [hep-th/0305191];
O.K. Kwon and P. Yi, *String fluid, tachyon matter and domain walls*, *J. High Energy Phys.* **09** (2003) 003 [hep-th/0305229];
W.S. L'Yi, *The energy-carrying velocity of the tachyon of unstable d- branes*, hep-th/0305249;
J. Kluson, *Particle production on half s-brane*, hep-th/0306002;
L. Martucci and P.J. Silva, *Kinky D-branes and straight strings of open string tachyon effective theory*, *J. High Energy Phys.* **08** (2003) 026 [hep-th/0306295].

[34] M.R. Garousi, *Non-commutative world-volume interactions on D-branes and Dirac-Born-Infeld action*, *Nucl. Phys.* **B 579** (2000) 209 [hep-th/9909214];
M.R. Garousi, *Non-commutative world-volume interactions on D-branes and Dirac-Born-Infeld action*, *Nucl. Phys.* **B 579** (2000) 209 [hep-th/9909214].

[35] T. Mehen and M.B. Wise, *Generalized $\star$-products, Wilson lines and the solution of the Seiberg-Witten equations*, *J. High Energy Phys.* **12** (2000) 008 [hep-th/0010204];
T. Mehen and M.B. Wise, *Generalized $\star$-products, wilson lines and the solution of the Seiberg-Witten equations*, *J. High Energy Phys.* **12** (2000) 008 [hep-th/0010204].

[36] H. Liu and J. Michelson, *$\star$-Trek: the one-loop $N = 4$ noncommutative SYM action*, *Nucl. Phys.* **B 614** (2001) 279 [hep-th/0008205].

[37] H. Liu, *$\star$-trek II. $\star_n$ operations, open Wilson lines and the Seiberg-Witten map*, *Nucl. Phys.* **B 614** (2001) 305 [hep-th/0011125].

[38] S.R. Das and S.P. Trivedi, *Supergravity couplings to noncommutative branes, open Wilson lines and generalized star products*, *J. High Energy Phys.* **02** (2001) 046 [hep-th/0011131].

[39] Y. Okawa and H. Ooguri, *How noncommutative gauge theories couple to gravity*, *Nucl. Phys.* **B 599** (2001) 55 [hep-th/0012218]; *How noncommutative gauge theories couple to gravity*, *Nucl. Phys.* **B 599** (2001) 55 [hep-th/0012218].

[40] H. Liu and J. Michelson, *Supergravity couplings of noncommutative D-branes*, *Nucl. Phys.* **B 615** (2001) 169 [hep-th/0101016].

[41] Y. Okawa and H. Ooguri, *Energy-momentum tensors in matrix theory and in noncommutative gauge theories*, hep-th/0103124.

[42] S. Mukhi and N.V. Suryanarayana, *Gauge invariant couplings of noncommutative branes to Ramond-Ramond backgrounds*, *J. High Energy Phys.* **05** (2001) 023 [hep-th/0104045].

[43] M.R. Garousi, *Dirac-Born-Infeld action, Seiberg-Witten map and Wilson lines*, *Nucl. Phys.* **B 611** (2001) 467 [hep-th/0105139].

[44] M. Green, J. Schwarz and E. Witten, *Superstring theory*, vol. 1, Cambridge University Press, Cambridge 1987.

[45] J. Polchinski, *String theory*, Cambridge University Press, Cambridge 1998.

[46] C. Bachas, *D-brane dynamics*, *Phys. Lett.* **B 374** (1996) 37 [hep-th/9511043];
A.A. Tseytlin, *Self-duality of Born-Infeld action and Dirichlet 3-brane of type-IIB superstring theory*, *Nucl. Phys.* **B 469** (1996) 51 [hep-th/9602064].

[47] E.S. Fradkin and A.A. Tseytlin, *Nonlinear electrodynamics from quantized strings*, *Phys. Lett.* **B 163** (1985) 123;
A. Abouelsaood, C.G. Callan Jr., C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599;
E. Bergshoeff, E. Sezgin, C.N. Pope and P.K. Townsend, *The Born-Infeld action from conformal invariance of the open superstring*, *Phys. Lett.* **B 188** (1987) 70;
R.R. Metsaev, M.A. Rakhmanov and A.A. Tseytlin, *The Born-Infeld action as the effective action in the open superstring theory*, *Phys. Lett.* **B 193** (1987) 207;
A.A. Tseytlin, *Renormalization of mobius infinities and partition function representation for string theory effective action*, *Phys. Lett.* **B 202** (1988) 81;
R.G. Leigh, *Dirac-Born-Infeld action from Dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[48] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of Möbius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205; *Generating functional for scattering amplitudes and effective action in the open superstring theory*, *Phys. Lett.* **B 207** (1988) 157; *Two loop beta function in the open string sigma model and equivalence with string effective equations of motion*, *Mod. Phys. Lett.* **A 3** (1988) 1349;
Y. Kitazawa, *Effective lagrangian for open superstring from five point function*, *Nucl. Phys.* **B 289** (1987) 599.

[49] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[50] A.A. Tseytlin, *On non-abelian generalisation of the Born-Infeld action in string theory*, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[51] D. Gross and E. Witten, *Superstring modifications of Einstein's equations*, *Nucl. Phys.* **B 277** (1986) 1

[52] A. Hashimoto and I.W. Taylor, *Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action*, *Nucl. Phys.* **B 503** (1997) 193 [hep-th/9703217];
F. Denef, A. Sevrin and J. Troost, *Non-abelian Born-Infeld versus string theory*, *Nucl. Phys.* **B 581** (2000) 135 [hep-th/0002180].

[53] A. Neveu and J. Scherk, *Connection between Yang-Mills fields and dual models*, *Nucl. Phys.* **B 36** (1972) 155;
J. Scherk and J.H. Schwarz, *Dual models for nonhadrons*, *Nucl. Phys.* **B 81** (1974) 118;
A.A. Tseytlin, *Vector field effective action in the open superstring theory*, *Nucl. Phys.* **B 276** (1986) 391.

[54] E. Coletti, I. Sigalov and W. Taylor, *Abelian and nonabelian vector field effective actions from string field theory*, *J. High Energy Phys.* **09** (2003) 050 [hep-th/0306041].

[55] M.R. Garousi, *On-shell S-matrix and tachyonic effective actions*, *Nucl. Phys.* **B 647** (2002) 117 [hep-th/0209068].

[56] M.R. Garousi, *Off-shell extension of S-matrix elements and tachyonic effective actions*, *J. High Energy Phys.* **04** (2003) 027 [hep-th/0303239].

[57] M.R. Garousi, *Slowly varying tachyon and tachyon potential*, *J. High Energy Phys.* **05** (2003) 058 [hep-th/0304145].

[58] S. Corley, D.A. Lowe and S. Ramgoolam, *Einstein-Hilbert action on the brane for the bulk graviton*, *J. High Energy Phys.* **07** (2001) 030 [hep-th/0106067];
F. Ardalan, H. Arfaei, M.R. Garousi and A. Ghodsi, *Gravity on noncommutative D-branes*, *Int. J. Mod. Phys.* **A 18** (2003) 1051 [hep-th/0204117].

[59] A.A. Tseytlin, *Sigma model approach to string theory effective actions with tachyons*, *J. Math. Phys.* **42** (2001) 2854 [hep-th/0011033].

JHEP12(2003)036